(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972 **Roll No.**

B.Tech.

(SEM VI) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

GRAPH THEORY

Time : 3 Hours *Total Marks : 100*

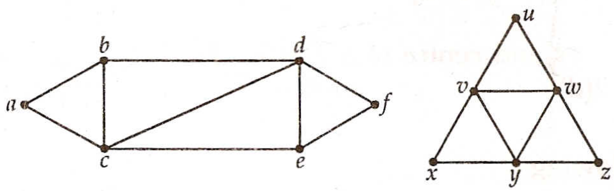
Note : *(i) Attempt **all** the questions.*

*(ii) All questions carry **equal** marks.*

1. Attempt **any four** parts of the following : **(4x5=20)**

(a) Define the degree of a vertex in a graph. Prove that the sum of the degrees of all vertices of a graph in a graph is twice the number of edges in graph.

(b) Define isomorphism of graphs. For the following pair of graphs, determine whether or not the graphs are isomorphic. Explain your answer.



(c) Prove that a simple graph with n vertices and k components can have atmost $(n-k)(n-k+1)/2$ edges.

(d) Discuss travelling sales man problem.

(e) Define the following with one example.

(i) Complete graph

(ii) Eulerian graph

(iii) Hamiltonian graph

(iv) Bi-partite graph

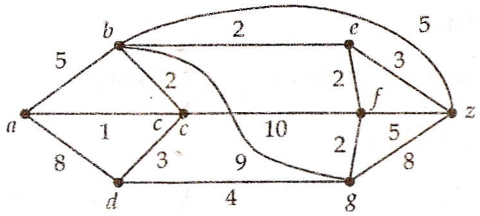
(v) Cut points of a graph

2. Attempt **any four** parts of the following : **(4x5=20)**

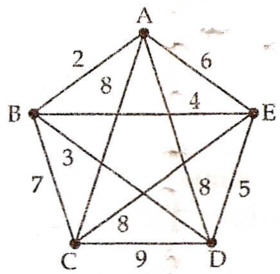
(a) If G is a non-trivial tree, then prove that G contains at least two vertices of degree 1.

(b) Define binary trees and discuss two important applications of it.

(c) Apply Dijkstra algorithm to find out the shortest path from the vertices a to z in the following graph.



(d) Use Prims algorithm to find out the minimal spanning tree of the following graph.



(e) Define fundamental circuits. Find the sets of fundamental circuits (four only) of the graph given in Q. No. 2(d). Take any spanning tree and find it corresponding to that spanning tree.

(f) Define eccentricity of the vertex and centre of a graph. Find the centre of the graph given in question no. 2(d).

3. Attempt **any four** parts of the following : **(4x5=20)**

(a) Define a planar graph. State and prove the Euler's formula for planar graph.

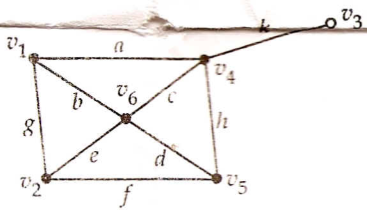
(b) Define edge and vertex connectivity of a graph. Prove that the vertex connectivity of any graph will never be more than the edge connectivity.

(c) Show that the Kuratowski's first (K_5) and second ($K_{3,3}$) are nonplanar graphs.

(d) Show that a graph has a dual if and only if it is planar.

(e) Define the thickness of a graph, give one example. Find the thickness of Kuratowski's first and second graphs.

(f) Define cut-sets. List all cut-sets with respect to the vertex pair v_2, v_3 in the following graph.



4. Attempt **any two** parts of the following : **(2x10=20)**

(a) Define basis vectors of a graph. Show that the number of distinct basis possible in a cut-set subspace is :

$$\frac{1}{r!}\left(2^r-2^0\right)\left(2^r-2^1\right)\left(2^r-2^2\right)......\left(2^r-2^{r-1}\right)$$

(b) Find the relationship among reduced incidence matrix A_f, fundamental circuit matrix B_f and fundamental cut set matrix C_f of a connected graph. Also establish the relation by giving one example.

(c) (i) If B is a circuit matrix of a connected graph G, with e edges and n vertices, then show that the rank of B is equal to the nullity of G.

(ii) Prove that the rank of a cut-set matrix is equal to the rank of the graph.

5. Attempt **any two** parts of the following : **(2x10=20)**

(a) Prove that an m-vertex graph is a tree if and only if its chromatic polynomial is :

$$P_m(\lambda) = \lambda\,(\lambda-1)^{m-1}$$

(b) Show that the number of simple labelled graph of n vertices is $2^{n(n-1)/2}$.

(c) Define indegree and outdegree of a vertex of a directed graph. Prove that for a directed graph D with n vertices $(v_1, v_2, ...v_n)$ and q edges.

$$\sum_{i=1}^{n} \text{in degree}\,(v_i) = \sum_{i=1}^{n} \text{outdegree}\,(v_i) = q$$

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