Printed Pages—3 **ECS505**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 **Roll No.** | | | | | | | | | | |

B. Tech.

(SEM. VI) THEORY EXAMINATION 2010-11

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

Note :—(1) Attempt **ALL** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(3×4=12)**

(a) What is a bipartite graph ? Obtain expression for the maximum number of edges in a bipartite graph.

(b) When are two graphs said to be isomorphic ? Show that two graphs need not be isomorphic even when they both have the same order and same size.

(c) Define the Hamiltonian graph. Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit.

(d) Discuss the travelling salesman problem.

(e) In a park, jogging track is designed in such a way that there are four end points (say N, E, W, S). End point W is connected by two paths from end points N and S each and by single path from end point E. End points N and E are connected by single path. End points S and E are also connected by single path. Show that a jogging person can't return to its starting end point after walking through all the paths exactly once.

(f) Prove that if a connected graph G is decomposed into two subgraphs g_1 and g_2, there must be at least one vertex common between g_1 and g_2.

2. Attempt any **two** parts of the following : 6×2=12

(a) Show that :

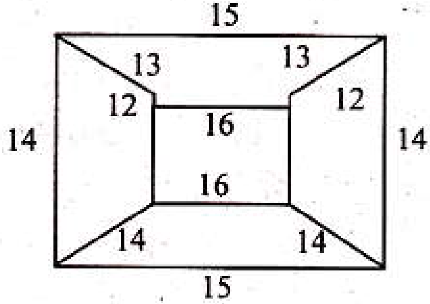
(i) If a graph G have one and only one path between every pair of vertices, G is tree.

(ii) The number of terminal vertices in a binary tree with n vertices is $(n + 1)/2$.

(b) (i) Define the terms : Metric and Fundamental circuit.

(ii) Prove that the nullity of a graph does not change when you either insert a vertex in the middle of an edge, or remove a vertex of degree two by merging two edges incident on it.

(c) Find all the minimum spanning trees in the following graph using Prim's algorithm :



3. Attempt any **two** parts of the following : **(6×2=12)**

(a) Define the edge-connectivity and vertex connectivity of a graph. Prove that for any graph :

$$k(G) \le \lambda(G) \le \delta(G).$$

Where $k(G)$, $\lambda(G)$, $\delta(G)$ are connectivity number, edge connectivity number and minimum degree among the vertices in a graph respectively.

(b) Describe an algorithm to detect the planarity of a graph. Detect planarity of $K_{3,3}$.

(c) Define the thickness and crossing number of a graph. Find the thickness and crossing number of the complete graph with n vertices, where $n \le 8$.

4. Attempt any **four** parts of the following : **(3.5×4=14)**

(a) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge-disjoint union of circuits.

(b) Define the cut set subspace of connected graph. What is meant by dimension of a subspace ?

(c) Define a circuit vector and a cut set vector of a connected graph. Prove that a circuit vector and a cut set vector are orthogonal to each other w.r.t. mod 2 arithmetic.

(d) Sketch a graph G that has the following vectors (among others) in its circuit subspace : (0, 1, 1, 1, 1, 0, 0, 1), (0, 1, 1, 1, 0, 1, 1, 0), (0, 1, 0, 0, 1, 0, 1, 0), (0, 1, 0, 0, 0, 1, 0, 1), (1, 0, 1, 0, 1, 1, 0, 1), (1, 0, 1, 0, 0, 0, 1, 0), (1, 0, 0, 1, 1, 1, 1, 0) and (1, 0, 0, 1, 0, 0, 0, 1).

(e) Find chromatic polynomial $P(G, x)$, where G is a cyclic graph with n vertices where $n = 3$ or $n = 4$.

(f) Explain the covering and partitioning of a graph.