(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. VI) THEORY EXAMINATION 2011-12

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

1. Attempt any ***four*** parts of the following :— **(3×4=12)**

   (a) Prove that sum of the degrees of the vertices of any finite graph is even.

   (b) Prove that a finite graph is bipartite if and only if it contains no cycles of odd length.

   (c) What are some applications where it is necessary to find the length of a longest simple path between two vertices in a weighted graph ?

   (d) What does it mean for two simple graphs to be isomorphic ? Give an example of two graphs that have the same number of vertices, edges, and degrees of vertices, but that are not isomorphic.

   (e) What does it mean for a graph to be connected ? What are the connected components of a graph ?

   (f) Explain the travelling salesman problem.

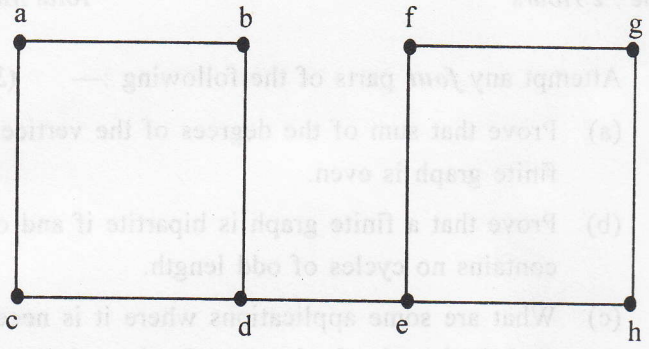
2. Attempt any ***two*** parts of the following :— **(2×6=12)**

   (a) Define an Euler circuit and an Euler path in an undirected graph. Describe the famous Königsberg bridge problem

and explain how to rephrase it in terms of an Euler circuit.

(b) Define a rooted tree and the root of such a tree. Draw a rooted tree with at least 10 vertices, where the degree of each vertex does not exceed 3. Identify the root, the parent of each vertex, the children of each vertex, the internal vertices, and the leaves.

(c) Draw all the spanning trees of the given graph ;



3. Attempt any *two* parts of the following :— **(2×6=12)**

(a) Define the edge connectivity and vertex connectivity of a graph. Show that a graph with n vertices and with vertex connectivity K must have at least Kn/2 edges.

(b) Show that a connected graph with n vertices and e edges has e – n + 2 regions.

(c) Define the following terms with suitable examples :—

(i) Geometrical dual of a graph

(ii) Thickness and crossings.

4. Attempt any *four* parts of the following :— **(3.5×4=14)**

(a) Differentiate between incidence and adjacency matrices.

(b) Give the relationship among reduced incidence matrix Af, fundamental circuit matrix Bj and fundamental cut set matrix Cf of a connected graph.

(c) Define the chromatic number of a graph. What is the chromatic number of the graph Kn when n is a positive integer ?

(d) State the four color problem. Are there graphs that cannot be colored with four colors ?

(e) Explore how the covering number of a graph G with n vertices is related to the diameter of G.

(f) State matching problem with suitable example.