(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 Roll No. | | | | | | | | | |

B.Tech.

(SEMESTER-VI) THEORY EXAMINATION, 2012-13

GRAPH THEORY

Time : 2 Hours] *[Total Marks : 50*

SECTION – A

1. Attempt **all** parts. $10 \times 1 = 10$

(a) Define bipartite graph with an example.

(b) Show that a connected graph with exactly two odd vertices is a universal graph.

(c) Prove that a connected graph G remains connected after removing an edge 'e' from G, if 'e' belongs to some circuit in G.

(d) Let G be a disconnected graph with n vertices, where n is even. If G has two components each of which is complete, prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.

(e) Define an Euler circuit and an Euler path in an undirected graph.

(f) Define the edge connectivity and vertex connectivity of a graph.

(g) Define the term : Metric and Fundamental Circuit.

(h) Show that number of terminal vertices in a binary tree with n vertices is $(n + 1)/2$.

(i) Give example of connected graph, that have lesser cut-vertices than bridges.

(j) Define rank and nullity of a graph.

SECTION – B

2. Attempt any **three** parts. **3 × 5 = 15**

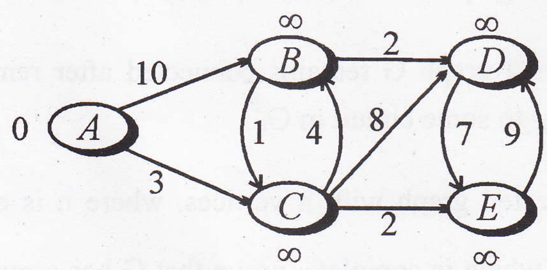
(a) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degrees.

(b) Show that for any graph : $k(G) \leq \lambda(G) \leq \delta(G)$, where k(G) is vertex connectivity, $\lambda(G)$ is edge connectivity and $\delta(G)$ is minimum degree of vertex.

(c) (i) Prove that every circuit has an even number of edges in common with any cutest.

(ii) Prove that a graph is connected if it has a spanning tree.

(d) Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to e.



SECTION – C

Attempt **all** parts. **5 × 5 = 25**

3. Attempt any **one** part :

(a) Show that, in the vector space of graph, the circuit subspace and the cutest subspace are orthogonal to each other.

(b) Let v be a cut-vertex of graph G, then $\bar{G} - v$ is connected. Where $\bar{G}$ is a complement of G. Prove it.

4. Attempt any **one** part :

(a) State and prove the Euler's formula for planar graph.

(b) What do you mean by a planar graph ? Draw a connected graph that has minimum degree greater than the number of bridges.

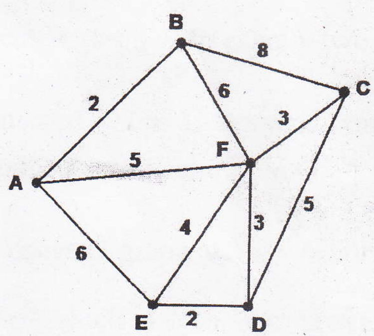
5. Attempt any **one** part :

(a) How many ways a tree on 5-vertices can be properly coloured with at most 4 colours ? Explain by taking an example of your own.

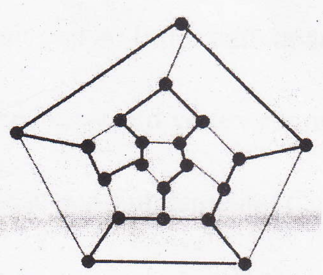
(b) Prove that "A tree is a connected graph without cycles".

6. Attempt any **one** part :

(a) Apply Prim's algorithm to design a minimum cost network represented by the graph :



(b) Find Hamilton's path & Hamilton cycle of the graph given below :



7. Attempt any **one** part :

(a) Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

(b) Define connectivity for directed and undirected graphs. Also, show that if 'a' and 'b' are the only two odd degree vertices of a graph G, then 'a' and 'b' are connected in G.
