

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2709

Roll No.

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**B.Tech.**

(SEM. VII) THEORY EXAMINATION 2011-12

**COMPUTATIONAL COMPLEXITY**

Time : 3 Hours

Total Marks : 100

**Note :—Attempt all questions.**

1. Attempt any **two** parts of the following : **(10×2=20)**
  - (a) What are the different models of computation ? Describe these models in comparative manner.
  - (b) Prove or disprove the following conjectures :
    - (i)  $f(n) = O(g(n))$  implies  
 $g(n) = O(f(n))$
    - (ii)  $f(n) + g(n) = \theta(\min(f(n), g(n)))$
    - (iii)  $f(n) = O(g(n))$  implies  
 $2^{f(n)} = O(2^{g(n)})$

where  $f(n)$  and  $g(n)$  are asymptotically positive functions.
  - (c) Are the following sets closed under the following operation.  
Prove your answer :
    - (i) FIN (the set of finite languages) under the function odds L, defined on languages as follows :  
 $\text{odds}(L) = \{w : \exists x \in L(w = \text{odds}(x))\}$ .
    - (ii) INF (the set of infinite languages) under the function odds L.

2. Attempt any **two** parts of the following : (10×2=20)

(a) What do you mean by complexity classes ? Discuss the relationship among the complexity classes.

(b) Show that if there is a reduction from  $P_1$  to  $P_2$  then :

(i) If  $P_1$  is undecidable then so is  $P_2$ .

(ii) If  $P_1$  is non-recursive enumerable then so is  $P_2$ .

(c) State and prove Rice theorem.

3. Attempt any **two** parts of the following : (10×2=20)

(a) Explain the general steps in establishing NP-completeness proof of a given problem.

(b) Explain the Gödel's incompleteness theorem with the help of examples.

(c) Write the randomized version of Quick sort algorithm.

4. Attempt any **two** parts of the following : (10×2=20)

(a) State the circuit satisfiability problem. Prove the circuit satisfiability problem belongs to the class NP.

(b) Consider the problem of multiplication of an  $n \times n$  matrix  $A = (a_{ij})$  by an  $n$ -vector  $x = (x_j)$ . The resulting  $n$ -vector  $y = (y_i)$  is given by the equation :

$$y_i = \sum_{j=1}^n a_{ij} x_j ; \text{ for } i = 1, 2, \dots, n.$$

Write an algorithm to perform matrix-vector multiplication by computing all the entries of  $y$  in parallel.

(c) Write short note on Interactive proofs.

5. Attempt any **two** parts of the following : (10×2=20)

(a) Explain the completeness and soundness properties of probabilistically checkable proof system.

(b) Explain the following class of problems :

(i) BPP

(ii) RP

(iii) CORP

(iv) ZPP.

(c) Write short note on Quantum computing.