(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 2753** **Roll No.** | | | | | | | | | | |

## B. Tech.

### (SEM. VII) THEORY EXAMINATION 2011-12

## DISCRETE STRUCTURES

*Time : 3 Hours* *Total Marks : 100*

**Note :–** (i) Attempt **all** questions.

(ii) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Let A and B be sets. Disprove the following :

(i) $A - B = B - A$

(ii) $A \times B = B \times A$

(b) How many different reflexive, symmetric relations are there on a set with three elements ?

(c) For the set of cities on a map, consider the relation x R y if and only if city x is connected by a road to city y. A city is considered to be connected to itself and two cities are connected even though there are cities on the road between them. Is this an equivalence relation or partial ordering ? Explain.

(d) Define the following functions on the integers by $f(k) = k + 1$, $g(k) = 2k$, and $h(k) = \lceil k/2 \rceil$. Which of these functions are one to one and which are onto ?

(e) Let p(n) be "$8^n - 3^n$ is a multiple of 5." Prove induction that p(n) is a tautology over N.

(f) Differentiate between proof by counter example and proof by cases methods.

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Discuss the connection between semigroups and monoids. Is every monoid a semigroup ? Is every semigroup a monoid ?

(b) Let V ={e, a, b, c}. Let * be defined (partially) by $x * x = e$ for all $x \in V$. Write a complete table for * so that [V,*] is a group.

(c) Prove that if a and b are elements of group G, then $(a * b)^{-1} = b^{-1} * a^{-1}$.

(d) When an element of a group is said to be a generator ? How many generators are there in the cyclic group of order 8 ?

(e) Find the inverse of the permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}$$

(f) If for a group G, $f: G \rightarrow G$ is given by $f(x) = x^2$, $x \in G$ and f is homomorphism, show that G is abelian.

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Let $D_{30}$ = {1, 2, 3, 5, 6, 10, 15, 30} and let the relation/ (divides) be a partial ordering on $D_{30}$.

(i) Find all lower bounds and upper bounds of 10 and 15.

(ii) Find the ***glb*** and ***lub*** of 10 and 15

(iii) Draw the Hasse diagram for $D_{30}$ with/ (divides)

(b) Define the distributive lattice. Prove that if $[L, \vee, \wedge]$ is a complemented and distributive lattice, then the complement of any element of lattice L is unique.

(c) Simplify the following Boolean expression using Karnaugh maps and draw the logic diagram of the simplified expression.

$F(A, B, C, D) = \Sigma(0,1, 3, 4, 7, 8, 10, 11, 12, 13, 15)$

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Construct the truth table for $x : (p \wedge (\sim q)) \vee (r \wedge p)$.

(ii) Give an example of a proposition other than x itself of a proposition generated by p, q, and r that is equivalent to x.

(iii) Give an example of a proposition other than x itself that implies x.

(b) What do you mean by valid argument ? Are the following arguments valid ? If valid, construct a formal proof; if not explain why.

For students to do well in discrete structure course, it is necessary that they study hard. Students who do well in courses do not skip classes. Students who study hard do well in courses. Therefore students who do well in discrete structure course do not skip class.

(c) Express the following statements using quantifiers :

(i) Every mathematics book that is published in India has a blue cover.

(ii) There exists a mathematics book with a cover that is not blue.

(iii) Every book with a blue cover is a mathematics book.

(iv) There are mathematics books that are published outside India.

(v) Not all books have bibliographies.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define the recurrence relation. Find the general solution of $S(K) - 3S(K-1) - 4S(K-2) = 4^K$.

(b) Draw the expression tree for the expression : $((a_3x + a_2)x + a_1)x + a_0$. Write the preorder, inorder, and postorder traversals of the resulting tree.

(c) Write short notes on any **three** of the following :

(i) Multi Graphs

(ii) Planar Graphs

(iii) Recursive algorithms

(iv) Pigeon hole principle.