(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2753 Roll No.

B.Tech.

(SEM. VII) ODD SEMESTER THEORY

EXAMINATION 2012-13

DISCRETE STRUCTURES

Time : 3 Hours *Total Marks : 100*

Note :– (1) Attempt **all** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) For any non empty sets A and B prove that

$$A \times B = B \times A \Leftrightarrow A = B.$$

(b) Let P be the set of all people. Let R be a binary relation on P such that (a, b) is in R if a is a brother of b. (Disregard half brothers and fraternity brothers.) Is R reflexive, Symmetric, Antisymmetric, Transitive ?

(c) Let R be a transitive and reflexive relation on A. Let T be a relation on A such that (a, b) is in T if and only if both (a, b) and (b, a) are in R. Show that T is an equivalence relation.

(d) What do you mean by inverse of a function ? Find the inverse of $f(x) = 5x - 7$.

(e) Show that $2^n > n^3$ for $n \geq 10$ by induction.

(f) Explain the recursively defined functions with a suitable example.

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Let (A, *) be a semigroup. Show that , for a, b, c in A, if a * c = c * a and b * c = c * b, then (a * b) * c = c * (a * b).

(b) Let (A, *) be a group. Show that (A, *) is an abelian if and only if $a^2 * b^2 = (a * b)^2$ for all a and b in A.

(c) Let G be a group. Show that each element a in G has only one inverse in G.

(d) Define subgroup. When a subgroup is said to be normal subgroup ? Explain with suitable example.

(e) What is a permutation group ? Give an example of a permutation group of order 6.

(f) Let G be the group of integers under the operation of addition and G' be the group of all even integers under the operation of addition. Show that the function $f : G \rightarrow G'$ defined by f(a) = 2a is an isomorphism.

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define a partial ordering. Show that divisibility relation on the set of positive integers is a partial order. Draw the Hasse diagram of the divisibility relation on the set {2, 3, 5, 9, 12, 15, 18}.

(b) (i) Define a lattice. Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not a lattice.

(ii) Prove that if a and b are elements in a bounded, distributive lattice and if a has a complement a', then

$$a \vee (a' \wedge b) = a \vee b$$

$$a \wedge (a' \vee b) = a \wedge b.$$

(c) Draw the circuit(gate) diagram of

$$f(x_1, x_2, x_3) = (x_1 \cdot x_2 + x_3) \cdot (x_2 + x_3) + x_3.$$

Simplify the function using basic Boolean algebra laws and also draw the logic diagram of the simplified function.

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Write a compound statement that is true when exactly two of the three statements p, q, r is true.

(ii) Show that $((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$ is a tautology.

(b) Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent. Show by using truth table as well as by developing a series of logical equivalences.

(c) (i) Show that $\sim\forall x\,(P(x) \rightarrow Q(x))$ is logically equivalent to $\exists x\,(P(x) \wedge \sim Q(x))$, where all quantifiers have the same nonempty domain.

(ii) Express the statements "Some students in this class has visited Varanasi" and "Every student in this class has visited either Allahabad or Varanasi" using predicates and quantifiers.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Consider the recurrence relation :

$$a_r = a_{r-1} - a_{r-2}.$$

(i) Solve the recurrence relation, given that $a_1 = 1$ and $a_2 = 0$.

(ii) Can you solve the recurrence relation if it is given that $a_0 = 0$ and $a_3 = 0$?

(iii) Repeat part (ii) if it is given that $a_0 = 1$ and $a_3 = 2$.

(b) Determine if the relation R = {(1, 7), (2, 3), (4, 1), (2, 6), (4, 5), (5, 3), (4, 2)} is a tree on the set A = {1, 2, 3, 4, 5, 6, 7}. If it is tree, what is the root and height ? If it is not a tree, make the least number of changes necessary to make it a tree and give the root and height.

(c) Write short notes on any **three** of the following :

(i) Planar Graphs

(ii) Generating function

(iii) Isomorphism of graphs

(iv) Pigeon hole principle.