(Following Paper ID and Roll No. to be filled in your Answer Book)

## PAPER ID : 2754

 Roll No.
## B.Tech.

(SEM. VII) ODD SEMESTER THEORY EXAMINATION 2012-13
THEORY OF AUTOMATA AND FORMAL LANGUAGES
Time: 3 Hours
Total Marks : 100
Note : (i) Attempt all questions.
(ii) All questions carry equal marks.
(iii) Notations/Symbols/Abbreviations used have usualmeaning.

1. Attempt any two parts of the following :
(a) Construct a minimum state automata equivalent to a FA whose transitions are given as follows :

| Present | Next State |  |
| :---: | :---: | :---: |
| State | Input <br> a | Input <br> $b$ |
| $\rightarrow q_{0}$ | $q_{1}$ | $q_{2}$ |
| $q_{1}$ | $q_{3}$ | $q_{8}$ |
| $q_{2}$ | $q_{4}$ | $q_{3}$ |
| $q_{3}$ | $q_{5}$ | $q_{3}$ |
| $q_{4}$ | $q_{4}$ | $q_{6}$ |
| $q_{5}$ | $q_{8}$ | $q_{6}$ |
| $q_{6}$ | $q_{7}$ | $q_{4}$ |
| $q_{7}$ | $q_{6}$ | $q_{5}$ |
| $q_{8}$ | $q_{8}$ | $q_{7}$ |

Given that $\mathrm{q}_{4}, \mathrm{q}_{5}$ and $\mathrm{q}_{8}$ are final states.
(b) Design finite automata (DFA) over $\Sigma=\{0,1\}$ with minimum number of states which accepts all the strings that end with 11 and contain 101 as substring.
(c) (i) Discuss the Chomsky hierarchy of the languages.
(ii) Write the regular expression for the language of all strings of 0 's and 1's in which do not contain substring 000 .
2. Attempt any two parts of the following :
(a) (i) Let $r_{1}$ and $r_{2}$ be regular. Simplify the following regular expression :

$$
r_{1}\left(r^{*}{ }_{1} r_{1}+r^{*}{ }_{1}\right)+r^{*}{ }_{1}+\left(r_{1}+r_{2}+r_{1} r_{2}+r_{2} r_{1}\right)^{*}
$$

(ii) Prove that every language defined by a regular expression is also accepted by some finite automata.
(b) Obtain the regular expression for the following finite automata having $\mathrm{q}_{0}$ as final state :

| Present | Next State |  |
| :---: | :---: | :---: |
| State | Input <br> a | Input <br> b |
| $\rightarrow \mathrm{q}_{0}$ | $\mathrm{q}_{3}$ | $\mathrm{q}_{1}$ |
| $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ | $\mathrm{q}_{0}$ |
| $\mathrm{q}_{2}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{3}$ |
| $\mathrm{q}_{3}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{2}$ |
| $\mathrm{q}_{4}$ | $\mathrm{q}_{5}$ | $\mathrm{q}_{4}$ |
| $\mathrm{q}_{5}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{3}$ |

(c) (i) If L and M are regular languages then $\mathrm{L}-\mathrm{M}$ is also regular language. Prove.
(ii) State the pumping lemma for regular expressions. Use the pumping lemma to prove that the language L is not regular. L is defined as follows :

$$
\mathrm{L}=\left\{(01)^{\mathrm{n}} \mid \mathrm{n} \text { is prime number }\right\} .
$$

3. Attempt any two parts of the following :
(a) (i) The set of context free languages is closed under intersection operation. Prove the statement or give counter example.
(ii) Determine whether following grammar is ambiguous or not?

$$
\mathrm{S} \rightarrow \mathrm{ictS}|\mathrm{ictSeS}| \mathrm{a} .
$$

(b) Simplify the following context free grammar $G$ to an equivalent context free grammar that do not have any useless symbol, null production or unit production:

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{~A}|\mathrm{~B}| \mathrm{C} \\
& \mathrm{~A} \rightarrow \mathrm{aAa} \mid \mathrm{B} \\
& \mathrm{~B} \rightarrow \mathrm{bB} \mid \mathrm{bb} \\
& \mathrm{C} \rightarrow \mathrm{aCaa} \mid \mathrm{D} \\
& \mathrm{D} \rightarrow \mathrm{baD}|\mathrm{abD}| \mathrm{aa}
\end{aligned}
$$

S is the start symbol.
(c) (i) Give an algorithm to decide whether language generated by a given CFG is finite.
(ii) Convert the following grammar into Greibach Normal Form (GNF).
$\mathrm{S} \rightarrow \mathrm{ABb} \mid \mathrm{a}$
A $\rightarrow$ aaA
$B \rightarrow b A b$
4. Attempt any two parts of the following :
(a) What is a Push Down Automata (PDA)? Construct a PDA which accepts the language $L$ given by $L=\left\{0^{n} 1^{2 n} \mid n\right.$ is non-negative integer\}.
(b) (i) Prove that if a PDA $M_{1}$ accepts language $L$ by final state then there exist a PDA $\mathrm{M}_{2}$ which accepts L by empty stack.
(ii) Construct a Push Down Automata which accepts the language generated by the following context free grammar having S as start symbol :

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{aSA} \mid \mathrm{a} \\
& \mathrm{~A} \rightarrow \mathrm{bB} \\
& \mathrm{~B} \rightarrow \mathrm{~b}
\end{aligned}
$$

(c) Obtain a context free grammar that generates the language accepted by the PDA M with following transitions :
$\delta\left(\mathrm{q}_{0}, 1, \mathrm{Z}_{0}\right)=\left\{\left(\mathrm{q}_{0}, \mathrm{XZ}_{0}\right)\right\}$
$\delta\left(\mathrm{q}_{0}, 1, \mathrm{X}\right)=\left\{\left(\mathrm{q}_{0}, \mathrm{XX}\right)\right\}$
$\delta\left(\mathrm{q}_{0}, 0, X\right)=\left\{\left(\mathrm{q}_{0}, \mathrm{X}\right)\right\}$
$\delta\left(\mathrm{q}_{0}, \in, \mathrm{X}\right)=\left\{\left(\mathrm{q}_{\mathrm{l}}, \in\right)\right\}$
$\delta\left(\mathrm{q}_{1}, \in, \mathrm{X}\right)=\left\{\left(\mathrm{q}_{\mathrm{l}}, \in\right)\right\}$
$\delta\left(\mathrm{q}_{1}, 0, \mathrm{X}\right)=\left\{\left(\mathrm{q}_{\mathrm{p}}, \mathrm{XX}\right)\right\}$
$\delta\left(\mathrm{q}_{1}, 0, \mathrm{Z}_{0}\right)=\left\{\left(\mathrm{q}_{1}, \mathrm{Z}_{0}\right)\right\}$
Given that $q_{0}$ is start state and $q_{1}$ is final state.
5. Attempt any two parts of the following :
(a) Define Turing machine. Design a Turing machine that accepts the language $L$ over $\{a, b\}$ defined as follows :
$L=\left\{w w \mid w \in(a+b)^{*}\right\}$.
(b) (i) Prove that recursively enumerable languages are closed under intersection operation.
(ii) What do you understand by undecidable problem? Prove that Halting problem of Turing machine is undecidable.
(c) (i) State post correspondence problem (PCP). Write the steps to construct a PCP instance, given an instance of Modified Post Correspondence Problem (MPCP).
(ii) Write short notes on various variants of Turing Machine.

