# (SEM. VII) ODD SEMESTER THEORY EXAMINATION 2013-14

## DISCRETE STRUCTURES

*Time : 3 Hours* *Total Marks : 100*

**Note :–** (i) Attempt **all** questions.

(ii) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Let A, B and C be nonempty sets; show that $(A \cap B) \cap C = A \cap (B \cap C)$.

(b) Let R be the binary relation defined as $R = \{(a, b) \in R^2 \mid a - b \leq 3\}$. Determine whether R is reflexive, symmetric, antisymmetric or transitive.

(c) Let I be the set of all integers. Let R be a relation on I, defined by $R = \{(x, y) \mid x - y \text{ is divisible by } 6\}$.

Show that R is an equivalence relation.

(d) Define the inverse of a function. Does the function $f(n) = 10 - n$ from the set of integers to the set of integers have an inverse ? If so, what is it ?

(e) Show by induction that any integer of $3^n$ identical digits is divisible by $3^n$. (For example, 222 and 777 are divisible by 3; 222, 222, 222 and 555, 555, 555 are divisible by 9.)

(f) Differentiate between proof by counter example and proof by cases methods.

2. Attempt any **four** parts of the following : **(5×4=20)**

(a) Let (A, *) be a commutative semigroup. Show that if $a * a = a$ and $b * b = b$, then $(a * b) * (a * b) = a * b$.

(b) Let (S, o) be monoid such that for every x in S, $x \circ x = e$, where e is the identity element. Show that (S, o) is an abelian group.

(c) Prove that if a and b are elements of group G, then $(a * b)^{-1} = b^{-1} * a^{-1}$.

(d) Define the subgroup. Explain the cyclic subgroup with an example.

(e) What is a Symmetric Group ? Give an example of symmetric group of order 6 and degree 3.

(f) Define an integral domain. Is (A, +, *) an integral domain, where A is the set of all integers, and + and * be the ordinary addition and multiplication operations on integers ?

3. Attempt any two parts of the following : **(10×2=20)**

(a) (i) Prove that if $(A, \leq)$ and $(B, \leq)$ are posets, then $(A \times B, \leq)$ is a poset, with partial order $\leq$ defined by $(a, b) \leq (a', b')$ if $a \leq a'$ in A and $b \leq b'$ in B.

(ii) Let $(P(A), \leq)$ and $(P(B), \leq)$ be posets, where $A = \{a, b\}$, $B = \{a\}$, and $\leq$ is the set inclusion operation. Draw the Hasse diagram of $(P(A) \times P(B), \leq)$.

(b) How does a partial order differ from a lattice ? Define the distributive lattice. Show that a linearly ordered poset is a distributive lattice.

(c) What is the relationship between Boolean functions and Boolean Expressions ? Use the Karnaugh map method to find a Boolean expression for the function f whose truth table is as follows.

| x | y | z | f(x,y,z) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

4. Attempt any two parts of the following : **(10×2=20)**

(a) What does it mean for two propositions to be logically equivalent ? Describe the different ways to show that two compound propositions are logically equivalent. Show in at least two different ways that the compound propositions $\sim p \vee (r \rightarrow \sim q)$ and $\sim p \vee \sim q \vee \sim r$ are equivalent.

(b) What do you mean by valid argument ? Are the following arguments valid ? If valid, construct a formal proof; if not explain why.

"Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high paying job. Therefore, someone in this class can get a high paying job."

(c) Express the following statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier.

(i) Some old dogs can learn new tricks.

(ii) No rabbit knows calculus.

(iii) Every bird can fly.

(iv) There is no dog that can talk.

(v) There is no one in this class who knows French and Russian.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define the generating function. Solve the difference equation : $a_n^2 + a_{n-1}^2 = 1$ given that $a_0 = 2$.

(b) Define the Binary Search Tree. Prove that the maximum number of vertices at level k of a binary tree is $2^k$ and that a tree with that many vertices at level k must have at least $2^{k+1} - 1$ vertices.

(c) Write short notes on any **three** of the following :

(i) Representation of Graphs.

(ii) Euler graph.

(iii) Recursive algorithms.

(iv) Pigeon hole principle.