EIT-071

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 113751

Roll No.

B. Tech.

(SEM. VII) (ODD SEM.) THEORY EXAMINATION, 2014-15

DISCRETE STRUCTURES

Time : **3** Hours] [Total Marks : **100**

Note : Attempt **All** questions.

1 Attempt any **four** parts : **(4×5=20)**

(i) Show that n^3+2n is divisible by 3 using mathematical induction ?

(ii) Determine whether each of the following function are bijective or not :

a. F: R -> R; $f(x)=(x^2+1)/(x^2+2)$

b. F: R -> R; $f(x)=x^5+1$

(iii) Let R be a Relation from set A to B and S be a relation from set B to C, then show that $(RoS)^{-1} = (S^{-1}oR^{-1})$

(iv) Show that $R=\{(a, b) \mid a \equiv b \pmod{m}\}$ is an equivalent relation on Z. Show also if $x_1 \equiv y_1$ and $x_2 \equiv y_2$ then $(x_1+x_2) \equiv (y_1+y_2)$.

(v) Let N={1,2,3...} and a relation is defined in $N \times N$ as follows: (a, b) is related to (c, d) iff ad = bc then show that whether R is a equivalence relation.

(vi) Composition function is commutative. Prove the statement or give counter example.

2 Attempt any **four** parts : **(4×5=20)**

(i) If for each a and b in a group G, $(ab)^2 = a^2b^2$. Show that G is abelian.

(ii) Define cyclic group with an example.

(iii) Prove that $(Z_6, +_6)$ is an abelian group of order 6. Where $Z_6 = \{0,1,2,3,4,5\}$.

(iv) State and prove Lagrange's theorem.

(v) Consider G = {0,1,2,3,4,5,6,7,8,9} under addition modulo 10. Find out order of each element of the group.

(vi) Explain Field with an example.

3 Attempt any **two** parts : **(2×10=20)**

(i) Simplify the Boolean expression

$f(w,x,y,z) = \sum m\,(0,2,4,5,8,14,15)$,

$d(w,x,y,z) = \sum m(7,10,13)$

(ii) Explain POSET and Lattice with an example.

(iii) Draw the Hasse Diagram for the following set under partial ordering: ({1, 2, 3, 4, 9, 36}, /). Define Maximal, minimal, greatest and least element of POSET. Find these elements in the Hasse diagram. Is it a Lattice ?

4 Attempt any **two** parts : **(2×10=20)**

(i) Check the validity of the following arguments using inference rules:

a. $(p \wedge q) \rightarrow r,\ (r \rightarrow q),\ (r \wedge q) \rightarrow (q \wedge r) \vdash (p \wedge q) \rightarrow (q \wedge r)$

b. $\sim p \wedge q,\ r \rightarrow p,\ \sim r \rightarrow s,\ s \rightarrow t \vdash t$

(ii) Prove the validity of the following argument using predicate calculus :
"Every living thing is a human being or an animal. Mohan is alive and he is not an animal. All human being have hearts. Hence, Mohan has a heart"

(iii) Show that $(P \oplus Q) \leftrightarrow ((P \wedge \neg Q) \vee (\neg P \wedge Q))$ is a tautology or contradiction or contingency?

5 Attempt any **two** parts : **(2×10=20)**

(i) Solve the given recurrence relation :
$a_n - 4a_{n-1} + 3a_{n-2} = 3n^2 - 3n + 1$

(ii) Explain Extended Pigeonhole Principle. What is the minimum number of students required in a class to be sure that atleast 5 will receive the same grade if there are four possible grades ?

(iii) Write a short note on the following :
a. Planar graph
b. Euler graph
c. Graph coloring.