\section*{Paper Id | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{8}$ | $\mathbf{1}$ | $\mathbf{9}$ |
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Roll No. $\square$

## B. TECH <br> (SEM-VII) THEORY EXAMINATION 2018-19 <br> INFORMATION THEORY AND CODING

Time : 3 Hours
Max. Marks : 100
Note : Be precise in your answer. In case of numerical problem assume data wherever not provided

## SECTION - A

1. Attempt all parts of the following questions:
$2 \times 10=20$
(a) What is Entropy? List the properties of Entropy.
(b) What is the minimum value of $\left(p_{1}, p_{2}, p_{3}, \ldots . . p_{n}\right)=H(p)$ as $p$ ranges over the set of $n-$ dimensional probability vector? Find all p's that which achieve this minimum.
(c) State Log-sum inequality.
(d) Define typical set and write its properties.
(e) Write the consequences of AEP.
(f) State Source Coding theorem.
(g) Show that the expected length L of any instantaneous D -ary code for a random variable X is greater than or equal to the entropy $\mathrm{H}_{\mathrm{D}}(\mathrm{X})$, that is $L \geq H_{D}(X)$, with equality if and only if $D^{-l_{i}}=p_{i}$
(h) What do you mean by Binary symmetric channel?
(i) Differentiate between block codes and covolutional codes.
(j) Given the $(5,4)$ even parity block code. Find the codewords corresponding to $i_{1}=$ (1011) and $i_{2}=$ (1010) ?

## SECTION B

2. Attempt any three parts of the following questions:
(a) For the systematic $(6,3)$ code with

$$
P=\left[\begin{array}{lll}
1 & 0 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{array}\right]
$$

Detect and correct the single error that occurred due to noise. Draw its syndrome calculation circuit.
(b) Explain soft-decision decoding with example. Also give benefits of soft decoding.
(c) What is channel? Classify channels into different groups. Explain each type briefly and also calculate the channel capacity of each type.
(d) Find the (a) binary and (b) ternary Huffman codes for the random variable X with probabilities $p=\left(\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}\right)$. Also calculate $L=\sum p_{i} l_{i}$ in each case.
(e) The convolutional encoder has the following two generator sequences each of length 3( the same as the constraint length $\mathrm{K}=3$ ):

1) Input-top adder output path

$$
\left(g_{0}^{(1)}, g_{1}^{(1)}, g_{2}^{(1)}\right)=(1,1,1)
$$

2) Input-bottom adder output path

$$
\left(g_{0}^{(2)}, g_{1}^{(2)}, g_{2}^{(2)}\right)=(1,0,1)
$$

The impulse response of either input-output path of the encoder is the same as the corresponding sequence of connections from the shift register to the pertinent adder, with a ' 1 ' representing a connection and a ' 0 ' representing no connection.

Find the following:-
(i) Draw the encoder diagram
(ii) Top and bottom output sequences for input sequence 10011.
(iii) Find the codeword for input message sequence 10011 using transform domain approach.

## SECTION C

## Attempt any one part of the following question:

$$
1 \times 10=10
$$

3. (a) What do you mean by relative entropy and mutual information? State the properties of relative entropy and mutual information.
(b) Given a binary channel shown in the figure below:

(i) Find the channel transition matrix.
(ii) Find $P\left(y_{1}\right)$ and $P\left(y_{2}\right)$ when $P\left(x_{1}\right)=P\left(x_{2}\right)=0.5$.
(iii) Calculate $\mathrm{H}(\mathrm{X}), \mathrm{H}(\mathrm{Y}), \mathrm{H}(\mathrm{Y} / \mathrm{X}), \mathrm{H}(\mathrm{X} / \mathrm{Y})$ and $\mathrm{I}(\mathrm{X} ; \mathrm{Y})$.

## Attempt any one part of the following question:

4. (a) State and prove Channel coding theorem.
(b) For the $(6,3)$ Hamming code, the parity check matrix H is given by

$$
H=\left[\begin{array}{llllll}
1 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1
\end{array}\right]
$$

(i) Construct the generator matrix.
(ii) Determine the codeword that begins with 110 .
(iii) If the received vector Y is (011010), then calculate the syndrome and find if there is any error in the received codeword.

Attempt any one part of the following question:
5. (a) Explain automatic repeat request schemes in detail.
(b) Explain optimal codes? Discuss bounds on optimal code length.

## Attempt any one part of the following question:

6. (a) A DMS X have five symbols $x_{1}, x_{2}, x_{3}, x_{4}$ and $x_{5}$ with $P\left(x_{1}\right)=0.4, P\left(x_{2}\right)=$ $0.19, P\left(x_{3}\right)=0.16, P\left(x_{4}\right)=0.15$ and $P\left(x_{5}\right)=0.1$
(i) Construct Shannon-Fano-Elias code for X and calculate the efficiency of the code.
(ii) Repeat for the Huffman code and compare the results.
(b) Explain in detail error detection and correction.

Attempt any one part of the following question:
7. (a) For a $(7,4)$ Hamming code the received sequence at the receiver is 10101111001. Find if the received sequence has error, if so then find bit sequence that was transmitted and also find the original message bits.
(b) For the encoder shown in the figure below:-

(i) Draw trellis structure for input 10011 .
(ii) Draw code tree for the input 10011.

