Printed Pages : 4 **TIC – 801**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0395 Roll No.

B. Tech.

(SEM. VIII) EXAMINATION, 2007-08

OPTIMAL CONTROL

Time : 3 Hours] *[Total Marks : 100*

Note : *(1) Attempt **all** questions.*
*(2) **All** questions carry **equal** marks.*
(3) Be precise in your answer.
(4) No second answer book will be provided.

1 Attempt any **four** of the following : **5×4=20**

(a) Define an 'Optimal control' problem. Also discuss its various components.

(b) Derive the 'Euler-Lagrange' equation for a 'Fixed-end' problem.

(c) Find the extremal of the functional

$$J(x) = \int_0^{\pi/4} \left(x_1^2 + \dot{x}_2^2 + \dot{x}_1 \dot{x}_2 \right) dt$$

The boundary conditions are

$$x_1(0) = 0, x_1(\pi/4) = 1, x_2(0) = 0, x_2(\pi/4) = -1.$$

(d) What is Pontrygin's Minimum Principle ? Discuss, how an optimal control input is obtained with a suitable problem, with this approach.

(e) Discuss the two basic principles of 'Dynamic Programming'.

(f) Explain the 'Steepest Descent' method for the numerical solution of Two-point Boundary Problem.

2 Attempt any **four** of the following : **5×4=20**

(a) Discuss the algorithm for the solution of 'Discrete-time Linear State Regulator' problem.

(b) It is desired to determine the control law that causes the plant

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - 2x_2 + u$$

to minimize the performance measure

$$J = 10x_1^2 + 1/2 \int_0^{0.6} \left(x_1^2 + 2x_2^2 + u^2\right) dt$$

Discretize the above system for the sampling interval T=0.2 sec and also determine optimal control.

(c) Find the optimal control law for the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} u$$

with the performance index

$$J = \int_0^{\infty} \left(x_1^2 + u_1^2 + u_2^2\right) dt$$

(d) Consider the second order system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

It is desired to find optimal control

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which minimizes the performance index

$$J = \int_0^\infty x_1^2 \, dt$$

(e) Discuss the solution of 'Output Regulator' and 'Tracking Control' problems using the results of 'Linear state regulator' problem.

(f) Use the minimum principle to show that it is necessary to use a Bang controller to drive the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

from an initial state $x_1 = 1, x_2 = 0$ to a zero final state if the input variable is constrained. Find such a control if $-1 \le u(t) \le 2$.

3 Attempt any **two** of the following: **10×2=20**

(a) Consider the problem of state reconstruction for the system described by the equations

$$\dot{x}(t) = x(t) + w(t)$$

$$y(t) = x(t) + v(t)$$

$$Q = 4, R = 0.5, P_0 = 0, t_0 = 0$$

Obtain the Kalman gain of the estimator.

(b) Consider the system

$$x(k+1) = 2x(k) + w(k)$$

$$y(k) = x(k) + v(k)$$

$$Q(k) = 1, R(k) = 0.5, \bar{x}^0 = 1, P_0 = 2$$

Find, $\hat{x}(k+1/k)$ for, $k = 1, 2$

Given, $y(1) = 1, y(2) = 2$.

(c) Determine the Kalman gains $K(k)$ for, $k = 1$ to 2 for the following estimation problem :

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + w(k)$$

$$y(k) = x_1(k) + v(k)$$

$$Q(k) = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \; R(k) = 1, \; P_0 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

4 Attempt any **two** of the following : **10×2=20**

(a) Give the block diagram for the implementation of 'Discretized PI controller'. Also explain the various operations involved in the implementation.

(b) Give the sampled-data model of the DC Motor Position Control system with DMC-105 board.

(c) What are 'Digital Signal Processors ? Discuss the block diagram 'TMS320 DSP' manufactured by Texas Instruments.

5 Attempt any **two** of the following : **10×2=20**

(a) Discuss the effects of Finite Word length and Quantization on 'Controllability' and 'Closed Loop Pole Placement'.

(b) Discuss the 'Transfer Characteristics' and block diagram representation of a Quantizer.

(c) Discuss the time delays in Microprocessor-based Control Systems.
