



Printed Pages : 7

TIC - 801

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0395

Roll No.

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**B. Tech.**

**(SEM. VIII) EXAMINATION, 2008-09**

**OPTIMAL CONTROL**

*Time : 3 Hours*

*[Total Marks : 100*

- Note :**
- (i) Attempt **all** questions.
  - (ii) All questions carry **equal** marks.
  - (iii) Be precise in your answer.
  - (iv) No second answer book will be provided.

1 Attempt any **four** of the following : **5×4=20**

- (a) Discuss the various performance measures to be optimized in an optimal control problem.
- (b) Show that the extremal of the functional :

$$J(x) = \int_0^{\pi/4} (\dot{x}^2 - x^2) dt$$



which satisfies the boundary conditions

$$x(0) = 0, x(\pi/2) = 1 \text{ is}$$

$$\text{is } x^*(t) = \sin t$$

- (c) Discuss the Hamiltonian approach for a variational calculus problem.
- (d) For the system

$$\dot{x} = u$$

With,  $|u| \leq 1$ , find the control which drives the system from an arbitrary initial state to the origin and minimizes

$$J = \int_0^{t_1} |u(t)| dt ; t_1 \text{ is free}$$

- (e) How Dynamic Programming does employ principle of optimality for solving the multistage decision process.
- (f) Draw and explain the flow chart of the steepest descent algorithm.

2 Attempt any **four** of the following : 5×4=20

- (a) Derive the Matrix Riccati equation for a continuous time regulator problem.



- (b) It is desired to determine the control law that minimizes the performance measure :

$$J = 1/2 \int_0^{t_1} (3x^2 + 1/4u^2) dt, t_1 \quad \text{is}$$

specified for a first order system with differential equation given as

$$\dot{x} = 2x(t) + u(t)$$

- (c) The linear discrete system

$$x_1(k+1) = x_1(k) + x_2(k)$$

$$x_2(k+1) = x_2(k) + u(k)$$

is to be controlled to minimize the performance index.

$$J = \sum_{k=0}^2 [4x_1^2(k) + u^2(k)]$$

Obtain the optimal control sequences

$[u(0), u(1), u(2)]$ ; the initial state is

$$x(0) = [1 \ 0]^T.$$



- (d) Find the optimal control law for the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

With performance index

$$J = \int_0^{\infty} (x_1^2 + u_1^2 + u_2^2) dt$$

- (e) Discuss the Minimum Time control of a Linear Time Invariant system.
- (f) What is sub-optimal control problem? Consider the second order system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

It is desired to find optimal control

$$u = -[k_1 \ k_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which minimizes the performance index

$$J = \int_0^{\infty} x_1^2 dt \text{ under the constraint that } k_1 = 1.$$



3 Attempt any **two** of the following : 10×2=20

(a) For the system described by the equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$y(t) = [1 \quad 0] x(t) + v(t)$$

$$Q = 0.5, R = 8, P_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}, t_0 = 0$$

Find the equation for the optimal estimator.

(b) For the time-invariant system :

$$\dot{x}(t) = x(t) + w(t)$$

$$y(t) = x(t) + v(t)$$

$$Q = 4\alpha, R = \alpha, \alpha = \text{constant}$$

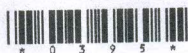
Find the time-invariant filter. Show that the answer is independent of  $\alpha$ .

(c) Consider the combined estimation and control problem for the system :

$$\dot{x}(t) = -x(t) + u(t) + w(t)$$

$$y(t) = x(t) + v(t)$$

$$Q = 4, R = 0.5, P_0 = 0, t_0 = 0$$



It is desired to find an optimal control law that minimizes the performance index

$$J = E \left\{ \frac{1}{2} x^2(2) + \frac{1}{2} \int_0^2 (2x^2(t) + u^2(t) dt) \right\}$$

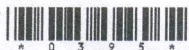
Find such a control and give a suitable scheme for the implementation of the control.

4 Attempt any **two** of the following : **10×2=20**

- (a) Draw and explain the block diagram of a microcomputer system with buses.
- (b) Discuss the microcomputer controlled DC motor system. Also draw the relevant diagrams.
- (c) Discuss the functional blocks of Galil DMC105 for a DC motor control system.

5 Attempt any **two** of the following : **10×2=20**

- (a) Discuss the advantages of Digital Signal Processors (DSPs) over Microprocessors. Discuss a DSP controlled motor system.



- (b) Discuss the effect of quantization of steady state error with a suitable example.
- (c) Explain how state variable technique is used to analyze the least bound error of a quantized system.

