PAPER ID-311584

Subject Code: KCS303

Roll No: | | | | | | | | | | | | | |

BTECH

(SEM III) THEORY EXAMINATION 2023-24

DISCRETE STRUCTURES & THEORY OF LOGIC

TIME: 3HRS **M.MARKS: 100**

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief.

Q no.	Question	Marks	CO
a.	Using predicate logic, express the statement "For all integers x and y, if x is divisible by y, then x is even."	2	
b.	If a proposition is satisfiable, does that mean it is a tautology? Explain your answer with an example.	2	
c.	Consider a cyclic group C_n of order n. Find the number of generators of C_{24}.	2	
d.	In a group G with order $\|G\|=30$, how many distinct cosets of a subgroup of order 5 can exist?	2	
e.	How many non-isomorphic simple graphs are there with 4 vertices?	2	
f.	In a graph with 9 vertices, each vertex having degree 4, how many different Hamiltonian cycles can be formed?	2	
g.	Classify the function g(x) = x^2 - 4x + 5 by its degree and type.	2	
h.	Express the Boolean function F(A, B, C) = Σ(0, 1, 2, 4, 5, 6) in its canonical form.	2	
i.	Define a relation R on set A = {1, 2, 3} such that R = {(1, 2), (2, 3), (1, 3)}. Determine the order of relation R.	2	
j.	Define a lattice (L, ≤) where L = {a, b, c, d} and the relation ≤ is defined such that a ≤ b, b ≤ c, and c ≤ d. Determine if (L, ≤) is a bounded lattice.	2	

SECTION B

2. Attempt any *three* of the following:

Q no.	Question	Marks	CO
a.	Construct a Hasse diagram for the partial order relation R on set A = {1, 2, 3, 4} such that R = {(1, 2), (2, 3), (1, 3), (2, 4), (3, 4)}.	10	
b.	Using Karnaugh maps, minimize the Boolean function F(A, B, C, D) = Σ(0, 2, 4, 5, 7, 10, 11, 15).	10	
c.	Determine whether the proposition $(P \vee \neg Q) \wedge (Q \vee \neg R) \wedge (R \vee \neg P)$ is satisfiable or not. If it is, provide a truth assignment that satisfies it.	10	
d.	Consider the ring of 2×2 matrices over the integers, denoted by $M_2(\mathbb{Z})$. Determine whether it is a field or not.	10	
e.	Prove that any planar graph with at least 3 vertices has a vertex of degree at most 5. Consider a planar graph with 20 vertices, each having degree 3. How many edges does this graph have?	10	

SECTION C

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3. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Let R = {(1, 2), (2, 3), (3, 4)}. Determine if the relation R is reflexive, symmetric, and transitive.	10	
b.	Define three relations R, S, and T such that R = {(1, 2), (2, 3)}, S = {(3, 4), (4, 5)}, and T = {(5, 6), (6, 7)}. Find the composite relation (R∘S)∘T.	10	

4. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Use De Morgan's laws to simplify the Boolean expression !(A AND B) OR !(C OR D).	10	
b.	Perform the Boolean operation A XOR (B OR C) for A = 1, B = 0, C = 1.	10	

5. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Consider the proposition $P \wedge (Q \vee R) \vee (\neg P \wedge \neg Q)$. Construct a truth table for this proposition and determine if it is a tautology.	10	
b.	Simplify the expression $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge \neg R)$ using laws of propositional logic.	10	

6. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Let G be a group of order 48. Prove or disprove: G must contain a subgroup of order 16	10	
b.	Let F be a field and R be a ring. Prove or disprove: If there exists a ring homomorphism $\phi: R \to F$, then R must be a field.	10	

7. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Prove using the Pigeonhole Principle that in any group of 6 people, there must be either 3 mutual friends or 3 mutual strangers.	10	
b.	Show that the chromatic number of a planar graph is at most 6, using the Four Color Theorem.	10	