



Roll No:

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BTECH
(SEM I) THEORY EXAMINATION 2023-24
MATHEMATICS-I

TIME: 3HRS

M.MARKS: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

Qno.	Question	Marks	CO
a.	Obtain the Eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$	2	1
b.	Define orthogonal matrix.	2	1
c.	State Lagrange's Mean value theorem	2	2
d.	Write statement of Leibnitz theorem.	2	2
e.	If $u = (x^2 + y^2 + z^2)$ then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$	2	3
f.	If $pv^2 = k$ and the relative error in p and v are respectively 0.05 and 0.025, show that the error in k is 10%.	2	3
g.	Evaluate $\int_0^1 \int_0^y dx dy$.	2	4
h.	Write the formula of Area and volume by integration.	2	4
i.	Find unit normal to the surface $z = 2xy$ at the point $(2,1,4)$ is ?	2	5
j.	Find the Value of K for which a vector $\vec{F} = (Kx + 4y^2z)i + (x^3 \sin z - 3y)j - (e^x + 4\cos x^2 y)k$ is a Solenoidal Vector.	2	5

SECTION B

2. Attempt any three of the following:

10x 3 = 30

a.	Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify Cayley Hamilton theorem.	10	1
b.	If $y = (\sin^{-1}x)^2$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2)y_n = 0$	10	2
c.	If u, v and w the roots of equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ , find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	10	3
d.	Apply Dirichlet's integral to find the Mass of an octane of the ellipsoid $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) + \left(\frac{z^2}{c^2}\right) = 1$, the density at any point being $\rho = kxyz$.	10	4
e.	Verify Gauss Divergence theorem for $\vec{F} = 4xzi - y^2j + yzk$ taken over the cube bounded by the planes: $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	10	5

SECTION C

3. Attempt any one part of the following:

10x 1 = 10

a.	Find the rank of matrix reducing into Canonical form for $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{bmatrix}$	10	1
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b.	Test the consistency and solve the following system of equations $2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3z + y - 4z = 0.$	10	1
4. Attempt any one part of the following:		10x 1 = 10	
a.	If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$	10	2
b.	Verify Rolle's theorem for the function $f(x) = x^2 - x + 1$ on $[0, 1]$.	10	2
5. Attempt any one part of the following:		10x 1 = 10	
a.	If $u_1 = \frac{x_2 x_3}{x_1}, u_2 = \frac{x_3 x_1}{x_2}, u_3 = \frac{x_1 x_2}{x_3}$, find $\frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)}$.	10	3
b.	Verify Euler theorem for $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$.	10	3
6. Attempt any one part of the following:		10x 1 = 10	
a.	Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$ by changing into polar Co-ordinates.	10	4
b.	Evaluate the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.	10	4
7. Attempt any one part of the following:		10x 1 = 10	
a.	Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational.	10	5
b.	Using Green's Theorem evaluate: $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$, where C is the square formed by the lines $x = \pm 1, y = \pm 1$.	10	5