Printed Pages : 7 TAS-204/MA-202(N)/MA-202(O)

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929/9919 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2006-07

MATHEMATICS - II

Time : 3 Hours] *[Total Marks : **100***

Note : *Attempt **all** the problems. The choice of problems is internal as indicated*

1 Attempt any **four** of the following : **5×4=20**

(a) A **4 kg** object falls from rest at time $t = 0$ in a medium offering a resistance in kg numerically equal to twice its instantaneous velocity in m/sec. Find the velocity and distance travelled at any time $t > 0$, and also the limiting velocity.

(b) Solve $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$.

(c) Solve $2\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 2y = e^x$.

(d) Solve $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 4\cot 2x$.

(e) Solve : $\frac{d^2x}{dt^2}+\frac{dy}{dt}+3x=e^{-t}$,

$$\frac{d^2y}{dt}-4\frac{dx}{dt}+3y=\sin 2t.$$

(f) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-\lambda^2y=0$

2 Attempt any **four** parts of the following : **5×4=20**

(a) Find the Laplace transform of

$$f(t)=\begin{cases} t^2 & 0<t<2 \\ t-1 & 2<t<3 \\ 7 & t>3 \end{cases}$$

(b) If $L\{f(t)\}=F(s)$, show that

$L\left\{\frac{1}{t}f(t)\right\}=\int_s^\infty F(s)ds$ Hence, find the Laplace transform of the function

$$f(t)=\int_0^t \frac{\sin\tau}{\tau}d\tau.$$

(c) Find the function whose Laplace transform is $\ln\left(1+\frac{1}{s}\right)$.

(d) State and prove convolution theorem for Laplace transform.

(e) Solve for $y(t)$ the equation

$$y(t) = 1 + \int_0^t y(\tau)\cos(t-\tau)\, d\tau.$$

(f) Solve, using Laplace transform method

$y''(t) + 4y'(t) + 4y(t) = 6e^{-t}$

$y(0) = -2 \quad y'(0) = 8'$

3 Attempt any **two** parts of the following : **10×2=20**

(a) Expand $f(x) = 0 < x < 2$ in a half range

(1) Sine series

(2) Cosine series

(b) If $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$ using half range cosine series expansion, show that

$$\frac{1}{1} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots = \frac{\pi^4}{96}.$$

(c) Solve $\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial x^2 \partial y} + 4\frac{\partial^3 u}{\partial y^3} = e^{x+2y}$.

OR

(1) Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| < 1 \\ -x, & |x| > 1 \end{cases}$$

(2) Find z-transform of $\cos \alpha k$ where $k \geq 0$

Note : Following question number **4** and **5** are for New Syllabus Only **(TAS-204/MA-202(New))**

4 Attempt any **two** parts of the following : **10×2=20**

(a) Find the solution(s) using the power series method $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2 y = 0$.

(b) Establish any **two** of the following recurrence formulae for the Legendre polynomials

(1) $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$

(2) $(x^2 - 1)\, P_n'(x) = n \left\{ x\, P_n(x) - P_{n-1}(x) \right\}$

(3) $(n+1)\, P_{n+1}(x) = (2n+1)x\, P_n(x) - n\, P_{n-1}(x)$

(c) Prove any **two** of the following recurrence formulae for the Bessel's functions $J_n(x)$:

(1) $2n\, J_n(x) = x\, (J_{n+1} + J_{n-1})$

(2) $J_{n+3}(x) + J_{n+5}(x) = \dfrac{2}{x}(n+4)\, J_{n+4}(x)$

(3) $x^2 J_r''(x) = (n^3 - n - x^2) J_n(x) + x\, J(x)_{n+1}$

5 Solve any **two** of the following : **2×10=20**

(a) Derive the one dimensional wave equation for vibrating string under suitable conditions.

(b) Describe the method of separation of variables for solving a partial differential equation. Hence solve the one dimensional heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial u}{\partial x}$$

under suitable initial and boundary conditions.

(c) Characterize the following partial differential equations into elliplic, parabolic and hyperbolic equations

$$A\frac{\partial^2 u}{\partial x^2}+2B\frac{\partial^2 u}{\partial x\,\partial y}+C\frac{\partial^2 u}{\partial y^2}=F(x,\ y,\ u,\ u_x, u_y)$$

Here $A,\ B,\ C$ may be functions of x and y.

Note : Following question Numbers **4** and **5** are for Old Syllabus only **(MA-202(Old).**

4 Attempt any **two** parts of the following : **10×2=20**

(a) Evaluate the integral by changing the order of integration

$$\int_0^{2a}\int_0^{\sqrt{2ay-y^2}}(x^2+y^2)dx\,dy.$$

(b) Evaluate $\iiint (x^2+y^2+z^2)dz\,dy\,dx$ over the volume enclosed by $x=0,\ y=0,\ z=0$ and the plane $x+y+z=p$.

(c) Apply the Dirichlet's integral to find the mass of a sphere $x^2+y^2+z^2=a^2$ where the density at any point being $\rho=Kx^2y^2z^2$.

5 Attempt any **two** parts of the following : **10×2=20**

(a) Solve by Cardon's method $8x^3 - 9x^2 + 1 = 0$.

(b) Fit a second degree parabola to the following data taking x as independent variable :

x	1	2	3	4	5	6	7	8	9
y	3	7	8	9	11	12	13	14	15

(c) Using the method of least square fit a linear relation of the form $P = a + bw$ for the following data :

$W(kg)$	50	70	100	120
$P(kg)$	12	15	21	25

P being the pull required to lift a weight ***W*** by pulley block. Estimate ***P*** when ***W*** is **150 kg** from this relation.