01236

Printed Pages—4 MCA—302

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1430 Roll No.

MCA

THIRD SEMESTER EXAMINATION, 2005-2006

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours *Total Marks : 100*

Note : *(i) Attempt **ALL** questions.*

(ii) All questions carry equal marks.

(iii) In case of numerical problems assume data wherever not provided.

(iv) Be precise in your answer.

1. Attempt ***any four*** of the following questions : **(5x4=20)**

(a) A recurrence $T(n) = 7T(n/2) + n^2$ describes running time of an algorithm A1. A competing Algorithm A2 has running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value of 'a' such that A2 is asymptotically faster than A1.

(b) Prove that $O(g(n)) \cap \omega(g(n))$ is empty set.

(c) Explain why the statement "The running time of algorithm A is at least $O(n^2)$" is meaningless.

(d) Given, $f(n) = \log n$, $g(n) = \sqrt{n}$. Show which function is asymptotically faster.

(e) Draw recursion tree for $T(n) = 4T(n/2) + cn$ where c is a constant and provide a tight asymptotic bound on its solution.

(f) Find the probability of a 4 turning up at least once in two tosses of a fair die.

2. Attempt *any four* of the following questions : **(5x4=20)**

(a) Prof. Murthy proposed to apply Quicksort to sort n distinct elements where he chooses the pivot elements randomly for partitioning the array. Analyse the run time behaviour of the algorithm. Will it outperform the case when median is chosen as pivot element.

(b) Design an algorithm to find the sum of largest $\log_2 N$ elements in an unsorted array of N distinct elements in O(n) time.

(c) As a function of minimum degree *t*, what is the maximum no of keys that can be stored in B-tree of height *h*.

(d) Say a node X is inserted into a R-B tree with RB-Insert and then immediately deleted with RB-Delete. Is the resulting R-B tree the same as the initial R-B tree ? Justify your answer.

(e) Find and prove the properties of binomial trees.

(f) Select the element with order - statistics = 6 in the given array (5,9,2,6,4,1,3,7), using selection algorithm. Show all the necessary steps.

3. Attempt *any two* of the following questions : **(10x2=20)**

(a) Which is a more efficient way to determine the optimal number of multiplications in a matrix-chain multiplication problem enumerating all the ways of parenthesizing the product and computing the number of multiplication for each, or running RECURSIVE-MATRIX-CHAIN ? Justify your answer. Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is (5,10,3,12,5,50,6).

(b) Suppose you are given a set $S=[a_1, a_2, \ldots\ldots a_n]$ of tasks, where each tasks ai requires unit time to process. You have one computer on which to run these tasks, and computer can run only one task at a time. Each tasks ai earns a profit pi, if it completes processing no later than its deadline di. Develop a greedy algorithm to schedule as many tasks as possible to earn maximum profit. Run your algorithm for n=5 and the following values :-

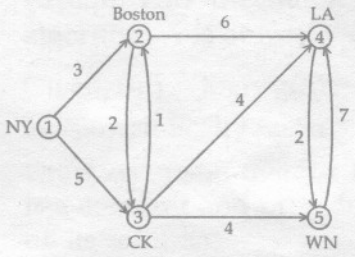
	i	1	2	3	4	5
Profit,	pi	50	20	15	30	45
Deadline	di	2	1	2	6	3

(c) A sequence of stack operations is performed on a stack whose size never exceeds K. After every K operations, a copy of the entire stack is made for backup purposes. Show that the cost of n stack operations, including copying the stack, is O(n) by assigning suitable amortized costs to the various stack operations.

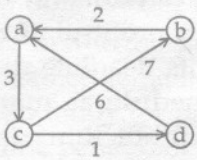
4. Attempt *any two* of the following questions : **(10x2=20)**

(a) Let G=(V,E) be an undirected graph represented using adjacency list. Design a linear time algorithm to search a given element starting from source node. Also return the shortest path to reach that element.

(b) Find the minimum distance of each station from New York . Give the algorithm used. Show all the steps and also analyse the algorithm to give its run-time



(c) Use Floydd-Warshall Algorithm to find the shortest paths for all the pairs of vertices in the given graph. Give the complexity of the algorithm.



5. Attempt *any two* of the following questions : **(10x2=20)**

(a) Prove vertex cover problem is NP-complete.

(b) (i) State and prove Chinese Remainder Theorem

(ii) Find all solutions to the equations

$35x \equiv 10 (\text{mod } 50)$.

(c) Write Rabin-Karp algorithm working modulo $9 = 11$, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 3141592653589793$ when looking for the pattern $P = 26$?

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