Printed Pages : 4 MCA302

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1430 Roll No.

M.C.A

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt **all** questions.

1 Attempt any **four** questions : 4×5=20

(a) Give an asymptotically tight bound $\left(i.e.\ \theta(\ )\right)$ on the summation $\sum_{k=1}^{n} K^r$, where $r \geq 0$ is a constant.

(b) Suppose $T_1(n) = O\left(f(n)\right)$ and $T_2(n) = O\left(f(n)\right)$ which of the following are true ? Justify

(i) $T_1(n) + T_2(n) = O\left(f(n)\right)$

(ii) $\dfrac{T_1(n)}{T_2(n)} = O(1)$

(iii) $T_1(n) = 0\left(T_2(n)\right)$

(c) Solve the average recurrence for quicksort.

(d) Prove that the height of a heap with n nodes is equal to $\lfloor \log_2 n \rfloor$.

(e) Illustrate the operation of counting-sort on the array $A = \langle 7,1,3,1,2,4,5,7,2,4,3 \rangle$.

(f) Modify bucket-sort algorithm to preserves it linear expected running time and makes its worst-case running time $O(n \lg n)$.

2 Attempt any **four** questions : **4×5=20**

(a) Insert items with the following keys (in the given order) into an initially empty binary search tree : 30, 40, 24, 58, 48, 26, 11, 13. Draw the tree after each insertion.

(b) Prove that the height of an AVL tree with n nodes is at most 1.4404 log n.

(c) Design an implementation of the following abstract data type with the set of operations :

insert (x, T) insert item x into the set T

delete (K, T) delete the K^{th} smallest element from T

member (x, T) return true if $x \in T$

all operations on an n item set are to take time $O(\log n)$.

(d) What is Fibonacci heap ? Illustrate the union process of two Fibonacci-heaps.

(e) Draw the 11-item hash table resulting from hashing the keys 12,44,13,88,23,94,11,39,20,16 and 5, using the hash function

$h(i) = (2i+5)$ mod 11 and assuming collisions are handled by linear probing.

(f) How many binary search trees are possible with n number of nodes.

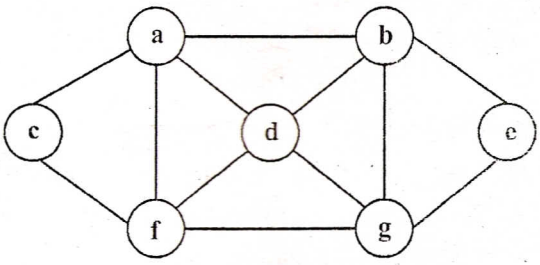
3 Attempt any **two** parts of the following : 10×2=20

(a) Design a dynamic programming algorithm for the change-making problem; given an amount n and unlimited quantities of coins of each of the denominations d_1, d_2, ..., d_m find the smallest number of coins that add up to n or indicate that the problem does not have a solution.

(b) A unit length closed interval on the real-line is an interval $[x, 1+x]$ describe an $O(n)$ algorithm that given input set $X = \{x_1, x_2, \ldots\ldots, x_n\}$ determines the smallest set of unit length closed intervals that contains all of the given points.

Assume $x_1 < x_2 < \ldots\ldots < x_n$.

(c) Apply back tracking to the problem of finding a Hamiltonian circuit in the following graph :

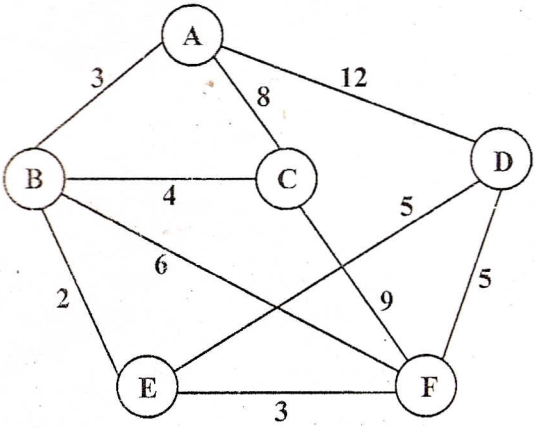


4 Attempt any **two** parts of the following : 10×2=20

(a) Solve the all-pairs shortest path problem for the diagraph with the weight matrix :

$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

(b) Discuss the Kruskas's algorithm and find the minimum cost spanning tree of the following graph :



(c) Show that, given a maximum flow in a network with m edges, a minimum cut of N can be computed in O(m) time.

Attempt any **two** parts of the following : **10×2=20**

(a) Draw a table representing the KMP failure function for the pattern string

" C G T A C G T T C G T A C "

(b) Prove that if $NP \neq CO - NP$ then $P \neq NP$.

(c) Write a nonrecursive version of algorithm. Euclid GCD and Extended Euclic GCD.
