

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7309

Roll No.

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MCA
(SEMESTER-III) THEORY EXAMINATION, 2012-13
DESIGN & ANALYSIS OF ALGORITHMS

Time : 3 Hours]

[Total Marks : 100

Note : The question paper contains **three** sections, Section – A, Section – B & Section – C with the weightage of **20, 30 & 50** marks respectively. Follow the instructions as given in each section.

Section – A

(Short answer type-2 marks each)

Note : Attempt **all** parts of this section :

2 × 10 = 20

1. (a) Differentiate between asymptotic notations O , Ω and Θ .
- (b) A recurrence relation for Tower of Hanoi problem (TOH) is $T(n) = 2T(n - 1) + 1$, with $T(1) = 1$ and $T(2) = 3$. Find the solution of recurrence relation for TOH problem.
- (c) Write all the 3 cases of master method to solve the recurrence $T(n) = aT(n/b) + f(n)$.
- (d) Show that any sorting algorithm based on comparison requires $\Omega(n \log n)$ comparisons.
- (e) Define a red-black tree. Show that a red-black tree with n internal nodes has height at most $2 \log(n + 1)$.
- (f) Enumerate the properties of a Binomial tree.
- (g) Consider a text $T[1.....50]$ of length 50 and a pattern $P[1....6]$ of length 6. Find the total number of comparisons (in worst case) to find a pattern P in a given text T ?
- (h) What is a Hamiltonian cycle?
- (i) Define P, NP and NP-complete class of problem. Write three problems which are NP-complete.
- (j) Explain randomized algorithm with the help of an example.

Section – B

(Medium answer questions 10 marks each)

Note : Attempt any **three** questions out of **five** questions from the following : $3 \times 10 = 30$

2. (a) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Using the basic definition of O , Ω , and Θ , prove / disprove the following :

(i) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$

(ii) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$

(b) Use a Recursion tree / Master method to give an asymptotic tight solution to the recurrence

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$$

3. (a) Illustrate the operation of COUNTING-SORT on the array $A = \langle 2, 5, 3, 0, 2, 3, 0, 3 \rangle$. Prove that counting sort is stable.

(b) What is amortized analysis ? Explain accounting method with the help of an example.

4. (a) Write an algorithm to sort the given array of element using Quick-sort. Illustrate the operation of PARTITION procedure on the array $A = \langle 2, 8, 7, 1, 3, 5, 6, 4 \rangle$. Take 'element 4' as the pivot element.

(b) Write a pseudo code for the following :

(i) $RB_Insert_Fixup(T, x) : \dots$ For inserting node x in a Red-Black tree.

(ii) $BINOMIAL_HEAP_UNION(H1, H2)$

5. (a) Explain backtracking method for solving. Draw a state space search tree to solve a 4-queens problem.

(b) Write a pseudo code for computing Longest Common Subsequence (LCS). Determine LCS for the sequence $X = \langle A, B, C, B, D, C, B \rangle$ and $Y = \langle B, D, C, A, B, A \rangle$

6. (a) Explain Kurshkal's algorithm using suitable example.

(b) Find all solutions to the following equations for x :

$$35x \equiv 10 \pmod{50}$$

Section – C

Note : Attempt any **one** parts from each question. **All** questions are compulsory : $10 \times 5 = 50$

7. (a) Draw the recursion tree for $T(n) = 4T(\lfloor n/2 \rfloor) + n$ and provide tight asymptotic bound on its solution.
- (b) Use Strassen's matrix multiplication algorithm to multiply the following two matrices :

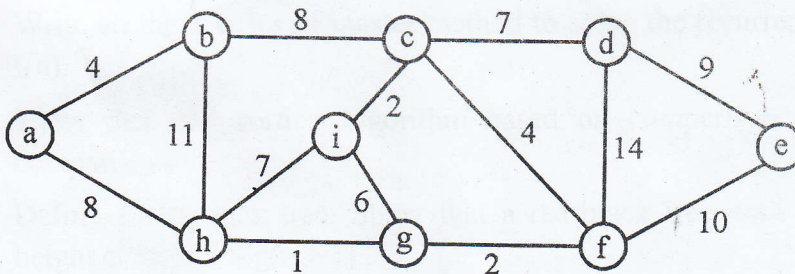
$$A = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

What is its time complexity ?

8. (a) Write an algorithm to sort the given array of element using Heap-sort i.e. HEAPSORT (A). Illustrate the operation of HEAPSORT on the array $A = \langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$. Analyze its running time in worst case also ?

- (b) What is "Greedy algorithm" for solving any problem ? Write an algorithm to find Minimum cost spanning tree (MCST) using Prim's algorithm. Analyze its time complexity also ?

9. (a) Are the Minimum spanning tree of any graph is unique ? Find MCST for the following using Prim's Algorithm, ('a' is a starting vertex)



- (b) Discuss traveling salesman problem with a suitable example.

10. (a) Given two polynomials $A(x)$ and $B(x)$ of degree bound n . Explain the process of FFT method for multiplying $A(x)$ and $B(x)$ in $\Theta(n \log n)$ time. Write its pseudo code (i.e. Recursive algorithm) for FFT.

- (b) Write an algorithm for the following :

Union-Find algorithm (i.e. union by rank) for Disjoint-set data structure.

11. (a) Draw the standard TRIE for the following set of strings :

{abab, baba, cccc, bbaaa, caa, bb aacc, cbcc, cbca}

(b) State Chinese Remainder theorem. Solve the following equations for x .

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{7}$$

$$x \equiv 6 \pmod{9}$$

