

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID : 214104

Roll No.

MCA

(SEM. I) THEORY EXAMINATION, 2015-16

DISCRETE MATHEMATICS

[Time:3 hours]

[Maximum Marks:100]

Section-A

Q.1 Attempt all questions from this section. (10x2=20)

- (a) Define power set.
- (b) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 3x + 7$  is a one one onto function, find  $f^{-1}$ .
- (c) In a group  $(G, *)$ , for any element  $a \in G$  prove that  $(a^{-1})^{-1} = a$ .
- (d) Define abelian group.
- (e) Draw the Hasse diagram of  $\langle D_{30}, \mid \rangle$ , where  $D_{30}$  is the set of all the divisors of 30.
- (f) Define lattice.

- (g) Show that there are  $\frac{n(n-1)}{2}$  edges in a complete graph of  $n$  vertices.
- (h) Define binary search tree.
- (i) Show that  $(p \rightarrow q) = \sim(p \vee \sim q)$ .
- (j) Determine the truth value for each of the following statements. Assume  $x, y$  are elements of set of integers.

$\forall x \exists y \quad x + y \text{ is even}$

$\exists x \forall y \quad x + y \text{ is even}$

### Section-B

**Note:** Attempt any five questions from this section :

(5 x 10 = 50)

2. (a) If  $A = \{a, b, c, d, e\}$ ,  $B = \{a, c, e, g\}$  and  $C = \{b, e, f, g\}$  then prove the following :

(i)  $(A \cup B) \cap C \neq A \cup (B \cap C)$

(ii)  $(A - B) \cap (A - C) = A - (B \cup C)$

- (b) Let  $R$  be an equivalence relation over set of integers  $I$  defined as :

$R = \{(a, b) \mid a - b \text{ is divisible by } 5\}$ . Find the equivalence classes of set  $I$ .

3. Check whether the following functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  are one-one onto :

(i)  $f(x) = e^x$

(ii)  $f(x) = |x|$

4. Show that the  $n^{\text{th}}$  roots of unity form a commutative group with respect to ordinary multiplication.

5. Solve the following recurrence relation :

$$a_n - 4a_{n-1} + 4a_{n-2} = n+1; a_0 = 0, a_1 = 1, n \geq 2$$

6. (a) Show that  $(p \wedge (\sim p \vee q)) \vee (q \wedge \sim (p \wedge q)) \equiv q$ .

(b) Write converse, inverse and contrapositive of the following statements :

(i) If the teacher is absent, then some students do not complete their homework.

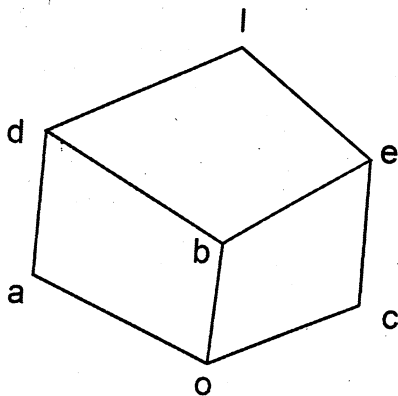
(ii) All the students complete their homework and the teacher is present.

7. Prove or disprove :

(a) Every simple Euler graph with an even number of vertices has an even number of edges.

(b) Peterson's graph is Hamiltonian

8. Consider the lattice  $L$  given below :



- (i) Find all sub-lattices with 5 elements.
- (ii) Find atoms.
- (iii) Find complement of  $a$  and  $b$  if they exist.
- (iv) Is  $L$  distributive?
- (v) Is  $L$  complemented?

9. (a) In a Boolean algebra, if  $a = b$  then prove that :  
 $a.b' + a'.b = 0$
- (b) Draw the simplified network of  
 $f(x, y, z) = x.y.z + x.y'.z + x'.y'.z$
10. Write a short notes on the following :
- (a) Euler graph
  - (b) Hamiltonian graph
  - (c) Chromatic number
  - (d) Planar graph
  - (e) Regular graph

### Section-C

Note : Attempt any two questions from this section.

(15x2=30)

1. (a) In a shipment of 50 CDs 10 are defective. Determine
- (i) In how many ways we can select 35 CDs. 2
  - (ii) In how many ways we can select 35 non-defective CDs. 2
  - (iii) In how many ways we can select 35 CDs containing exactly 5 defective CDs. 2
  - (iv) In how many ways we can select 35 CDs containing at least 5 defective CDs. 2

- (b) If  $R^{-1}$  and  $S^{-1}$  are the inverses of relation  $R$  and  $S$  respectively, then prove that  $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$ .

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2. (a) Following table gives the value of the function  $f(x, y, z)$ . Find the corresponding function. Draw a simplified circuit diagram of the function. Also find the minterm normal form of  $f(x, y, z)$ . (3+4+3)

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- (b) Determine the validity of the following argument

:

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“If wages increases, there will be inflation. The cost of living will not increase, if there is no inflation. Wages will increase; therefore the cost of living will not increase”.

3. (a) Define Ring.

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(b) For all  $a, b, c$  of ring  $R$  show that :  $4 \times 2.5 = 10$

(i)  $a \cdot 0 = 0 \cdot a = 0$

(ii)  $(-a) \cdot b = -(a \cdot b) = a \cdot (-b)$

(iii)  $a \cdot (b - c) = a \cdot b - a \cdot c$

(iv)  $(-a) \cdot (-b) = a \cdot b$

—x—